T_1^{in}); relative temperature head. Indices: 1, tube space; 2, intertube space; w, wall; i, ice; in, out, parameters corresponding to the input and output of the heat exchanger (model).

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FLOW AND HEAT TRANSFER IN A NONSTEADY JET GENERATED BY LARGE-AMPLITUDE GAS OSCILLATIONS

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The problem of the velocity field in a jet formed by nonlinear oscillations in a tube is solved. Equations are derived that describe heat transfer to a body placed at the jet axis. These relationships are tested experimentally.

Jet flows of an incompressible fluid and a gas comprise one of the best developed fields of boundary-layer science. The importance of this branch of knowledge is due to the wide prevalence of jet flows in nature and engineering. An entire class of jet flows exists, however, that has hardly been investigated up to now, namely, nonsteady jets. Such jets can be produced using a generator of large-amplitude oscillations, which consists of a resonance tube, one end of which is open and communicates with the ambient medium, and at the other end of which a flat piston moves harmonically [1]. Upon the excitation of oscillations in the tube, a nonsteady jet is formed at its open end, with the amplitude of velocity oscillations in the jet reaching 160 m/sec near resonance. Another way of generating a high-velocity nonsteady jet has been described in [2].

The high amplitude of the velocity pulsations makes it possible to use the generated jet to investigate heat transfer between bodies and oscillating flows in a wide range of variation of the oscillatory Reynolds number Re_{osc} and the Strouhal number Sh. Heat transfer between bodies and this jet is also interesting because the velocity oscillations in it are anharmonic: the spectrum of velocity oscillations contains a constant component and a number of harmonics [3, 4].

In this paper we attempt to investigate the velocity field in a nonsteady jet generated at the open end of a pipe during nonlinear gas oscillations in it, as well as heat transfer for bodies (a cylinder, sphere, and disk) placed at the jet axis.

To solve the hydrodynamic part of the problem, we used the law of conservation of momentum [5], which in the case of a nonsteady jet takes the form

$$\frac{\partial}{\partial t}\int_0^\infty ru\,dr + \frac{\partial}{\partial x}\int_0^\infty ru^2\,dr = 0. \quad (1)$$

We assume that the axial velocity u can be represented as a sum $u = u_0 + u_1 + u_2$, where u_0 is the time-averaged component and u_1 and u_2 are the first and second harmonics, with $u_1 \approx u_0$ and $u_2 \ll u_1$. After substituting u into (1) and averaging over time, we obtain

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$$J_0 = 2\pi\rho \int\limits_0^\infty (u_0^2 + \langle u_1^2 \rangle + \langle u_2^2 \rangle)\, r dr = \text{const.} \tag{2a}$$

We subtract (2a) from (1) and use the fact that $u_2 \ll u_1$. We then have

$$\frac{\partial}{\partial t} \int\limits_0^\infty r u_1 dr + \frac{\partial}{\partial x} \int\limits_0^\infty 2u_0 u_1 r dr = 0, \tag{2b}$$

$$\frac{\partial}{\partial t} \int\limits_0^\infty r u_2 dr + \frac{\partial}{\partial x} \int\limits_0^\infty (u_1^2 - \langle u_1^2 \rangle + 2u_0 u_2)\, r dr = 0. \tag{2c}$$

We seek solutions to Eqs. (2) in the form

$$u_0 = \frac{1}{x} F(\alpha, \eta), \quad u_1 = \frac{1}{x} F(\beta, \eta)\, [f_1(x) \sin \omega t + f_2(x) \cos \omega t],$$
$$u_2 = \frac{1}{x} F(\gamma, \eta)\, [g\ (x) \sin 2\omega t + g_2(x) \cos 2\omega t], \tag{3}$$

where $\eta = r/\sqrt{\varepsilon x}$, ε is the coefficient of turbulent viscosity, and $F(\alpha, \eta)$ is the velocity profile. We take

$$F(\alpha, \eta) = 2\alpha^2 \left(1 + \frac{\alpha^2}{4} \eta^2\right)^{-2}, \tag{4}$$

i.e., the profile of a steady, self-similar jet [6]. We take the boundary condition for (3) in the form $u(x = x_0, r = 0) = u_m(a + b \cos \omega t + c \cos \omega t)$, and then $2\alpha^2 = a u_m x_0$, $2\beta^2 = b u_m x_0$, and $2\gamma^2 = c u_m x_0$. Here x_0 is the distance from the open end of the tube at which formation (detaching) of the jet occurs.

The substitution of (3) into (2) results in the system of equations

$$\omega_1 f_1 + \frac{df_2}{dy} = 0, \quad -\omega_1 f_2 + \frac{df_1}{dy} = 0, \quad \omega_2 g_1 + \frac{dg_2}{dy} + \lambda \frac{d}{dy} (f_2^2 - f_1^2) = 0,$$
$$-\omega_2 g_2 + \frac{dg_1}{dy} + 2\lambda \frac{d}{dy} (f_1 f_2) = 0, \tag{5}$$

where $\omega_1 = k_1 Sh_0$ and $\omega_2 = k_2 Sh_0$; $Sh_0 = \omega x_0/u_m$; $k_{1,2}$ and λ are numbers calculated from the equations

$$k_1 = \frac{(b-a)^2}{4ab} \left(a + b - \frac{2ab}{b-a} \ln \frac{b}{a}\right)^{-1}, \quad k_2 = 2k_1(a, c), \quad \lambda = \frac{1}{6} b k_2; \tag{6}$$

$y = (x/x_0)^2$ is the dimensionless coordinate. The boundary conditions for $f_{1,2}$ and $g_{1,2}$ follow from the conditions for u: $f_1(y = 1) = g_1(y = 1) = 0$, $f_2(y = 1) = g_2(y = 1) = 1$.

The solution of system (5) has the form

$$f_1 = \sin[\omega_1(y-1)], \quad f_2 = \cos[\omega_1(y-1)], \quad g_1 = (1-A)\sin[\omega_2(y-1)] +$$
$$+ A \sin[2\omega_1(y-1)], \quad g_2 = (1-A)\cos[\omega_2(y-1)] +$$
$$+ A\cos[2\omega_1(y-1)], \quad A = \frac{2\lambda\omega_1}{\omega_2 - 2\omega_1}.$$

From (6) it follows that $\omega_1/\omega_2 \approx c/b$, so that (2a) is valid to second order. Substituting (3) into (2a), for the mean momentum we obtain $J_0 = (8\pi/3)\rho\varepsilon u_m x_0(a + b/2 + c/2)$. The coefficient of turbulent viscosity, as in the case of a steady jet, is expressed by the equation [6] $\varepsilon = \sigma\sqrt{J_0/\rho}$, where σ is an empirical constant.

We write the resulting expression for the velocity:

$$u = \frac{1}{x} \{F(\alpha, \eta) + F(\beta, \eta)\cos[\omega t - \omega_1(y-1)] + F(\gamma, \eta)(1-A) \times$$
$$\times \cos[2\omega t - \omega_2(y-1)] + F(\gamma, \eta) A \cos[2\omega t - 2\omega_1(y-1)]\}. \tag{7}$$

TABLE 1. Results of a Fourier Analysis of Velocity Oscillograms, f ≈ 20 Hz

$\bar{x}$	Experiment			Theory		
	$\bar{a}$	$\bar{b}$	$\bar{c}$	$\bar{a}$	$\bar{b}$	$\bar{c}$
6	0,38	0,5	0,18	0,39	0,52	0,09
8,4	0,36	0,46	0,18	0,4	0,52	0,08
10,8	0,36	0,46	0,14	0,41	0,54	0,06
14,4	0,38	0,42	0,14	0,43	0,57	0,06

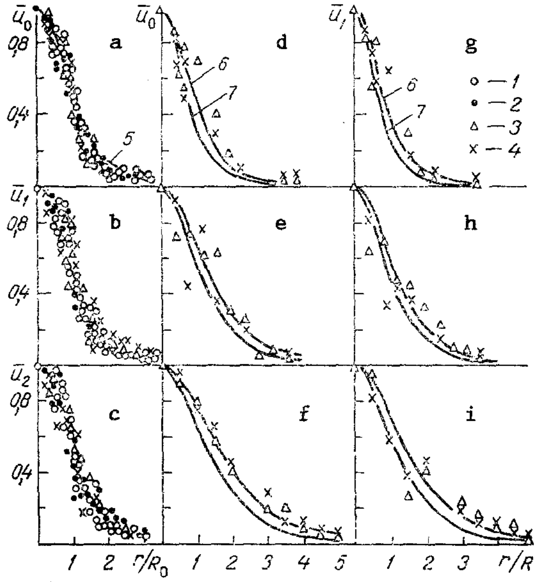

Fig. 1. Profiles of harmonic velocity components: universal profiles u_0 (a), $\bar{u}_1$ (b), and $\bar{u}_2$ (c); profiles of $\bar{u}_0$ at $\bar{x}$ = 10.8 (d), 16.05 (e), and 20.05 (e) and of $\bar{u}_1$ (g, h, i) (same $\bar{x}$ as for u_0); f = 16 Hz (1), 18 (2), 20 (3), and 22 Hz (4); 5) universal profile of a self-similar jet [6]; σ = 0.01 (6) and 0.012 (7).

For further calculations, we specify the constants a, b, and c in (7). From the results of [7] it follows that velocity oscillations in the jet may be described approximately by the function

$$u = u_m (m + \cos \omega t)\, \theta (m + \cos \omega t), \tag{8}$$

where $\theta(z) = 1$ at $z > 0$ and $\theta(z) = 0$ at $z < 0$; m is a constant that depends on the conditions of outflow at the open end of the tube. Near the fundamental resonance, m takes values from 0.4 to 0.5. Expanding (8) in a Fourier series, we obtain

$$a = \frac{1}{2\pi} [m (\pi + 2 \arcsin m) + 2 \sqrt{1 - m^2}],$$

$$b = \frac{1}{\pi} \left(\frac{\pi}{2} + \arcsin m + m \sqrt{1 - m^2} \right), \quad c = - \frac{2}{3\pi} (1 - m^2)^{3/2}. \tag{9}$$

Let us consider heat transfer for bodies in the jet. At velocities of about 150 m/sec, an oscillation frequency ~20 Hz, and bodies of size ~10^{-2} m, the Strouhal number is Sh ≪ 1, the oscillatory Reynolds number is $Re_{osc} \gg 1$, and for Pr ≈ 1 the heat transfer for a body should be quasi-steady [8] and occur in a boudnary layer that is formed. Consequently, the Nusselt number must satisfy $Nu = \langle Nu_s(u(t)d/\nu) \rangle$, i.e., the steady-state function Nu(Re) must be averaged over the velocity oscillation period.

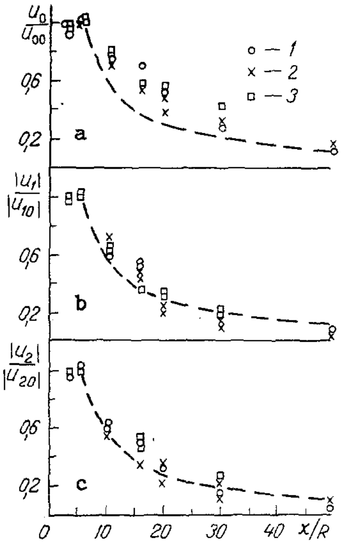

Fig. 2

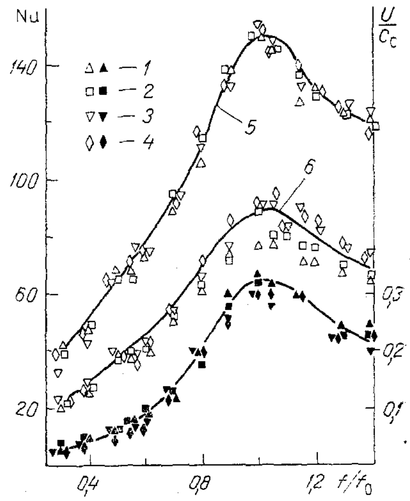

Fig. 3

Fig. 2. Dependecne of u_0 (a), $|u_1|$ (b), and $|u_2|$ (c) on $\tilde{x}$: f = 20 Hz (1), 22 (2), and 18 Hz (3).

Fig. 3. Frequency dependence of Nu (light points) and U (dark points): 1) $\tilde{x}$ = 3.5; 2) 6; 3) 8.5; 4) 11; 5) 0.02-m diameter sphere; 6) 0.01-m diameter cylinder.

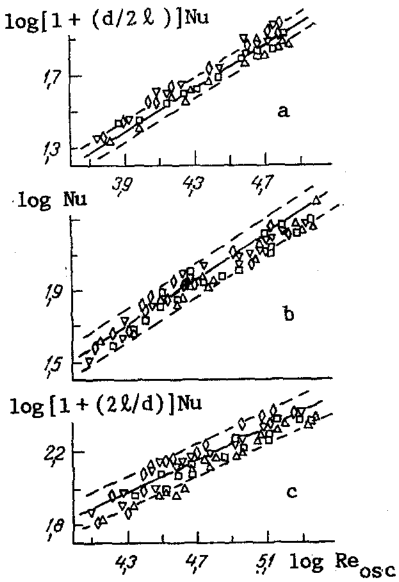

Fig. 4. Function $Nu(Re_{osc})$: a) cylinder; b) sphere; c) disk; notation same as in Fig. 3.

We use optimum data on heat transfer for a sphere and a cylinder in an established air stream. The following equation is given in [9] for a cylinder:

$$Nu = 0{,}197\, Re^{0.6}. \tag{10a}$$

For a sphere we use the McAdams equation [8]:

$$Nu = 0{,}33\, Re^{0.6}. \tag{10b}$$

Substituting (7) at r = 0 with the amplitudes (9) into (10) and averaging over the oscillation period, for m = 0.5 we obtain

$$\mathrm{Nu} = 0{,}1\,\mathrm{Re}_{\kappa}^{0,6}(x) \quad (11a)$$

for a cylinder and

$$\mathrm{Nu} = 0{,}17\,\mathrm{Re}_{\kappa}^{0,6}(x) \quad (11b)$$

for a sphere. Here the oscillatory Reynolds number is calculated for the range U(x) of velocity oscillations on the jet axis.

The experimental investigation of a jet and of heat transfer for bodies in the jet was carried out on apparatus that has been described in detail in [10]. The length of the resonator tube was 4 m and the resonance frequency was f_0 = 20 Hz. The velocity was measured with a constant-temperature thermoanemometer with subsequent computer linearization of the signal. The heat-transfer coefficient for bodies (a sphere, a cylinder, and a disk) placed at the jet axis was determined by the regular-regime method [11] using an automated measurement system. The models of the bodies were made of brass. The temperature was measured with a thermocouple mounted in the body, the signal from which was printed out after amplification. The cylinder axis was set perpendicular to the stream. The disk plane was also oriented perpendicular to the stream. The error in measuring the velocity did not exceed 20% and that for the heat-transfer coefficient did not exceed 10%.

In Table 1 we give experimental and theoretical values of the spectral components of the velocity signal for four values of $\bar{x}$. The agreement of the data for the constant component and the first harmonic up to $\bar{x} \approx 14$ must be considered satisfactory. The agreement is worse for the second harmonic, but since the amplitude of the second harmonic is only 20% of the maximum velocity amplitude, the error in determining it may be large. We note that $\bar{c}$ increases with decreasing m, as follows from (9): $\bar{c}$ = 0.12 for m = 0.4 at $x = x_0$, whereas $\bar{a}$ and $\bar{b}$ vary insignificantly.

Universal profiles of the velocity components are shown in Fig. 1a, b, c. The function for a steady self-similar jet from [6] is also shown for comparison. It is seen that the profiles of a nonsteady jet possess the universality property and practically coincide with the function from [6].

The velocity components as functions of r/R are shown in Fig. 1d-i. Taking σ = 0.01-0.012, we obtain good agreement between experiment and theory for the constant component and the first harmonic. For the second harmonic (not given in Fig. 1) we can only talk about qualitative agreement between the theoretical and experimental profiles.

The amplitudes of the velocity compnoents as functions of $\bar{x}$ are given in Fig. 2. The function $1/\bar{x}$ is also given there. As follows from a comparison of the theoretical and experimental data, the constant velocity compnent falls off slower than $1/\bar{x}$. This fact may be related to the influence on the flow of the finite initial gas flow rate through the jet cross section at $x = x_0$ [6]. In fact, our theoretical calculation is valid only for a jet with a zero initial flow rate, i.e., pulsating from a point source, as is easy to verify. The dependence of the amplitudes of the first and second harmonics on $\bar{x}$ agrees well with theory, however. From Fig. 2 one can conclude that the length of the section of formation of the jet is $x_0 \approx 6R$.

In Fig. 3 we show the results of measuring heat transfer for a sphere and a cylinder as a function of the frequency of oscillations excited in an oscillation generator. The results of measurements of the velocity range are also given here. It is seen that all the curves have a similar resonance nature, i.e., the Nusselt number is determined mainly by the amplitude of the velocity oscillations.

In Fig. 4 data on heat transfer are given as a function of the Reynolds number $\mathrm{Re}_{osc} = U(x)d/\nu$. All the experimental points are grouped around the functions

$$\mathrm{Nu} = 0{,}115\left(1 + \frac{d}{2l}\right)^{-1}\mathrm{Re}_{\kappa}^{0,6}(x) \quad (12a)$$

for a cylinder,

$$\mathrm{Nu} = 0{,}135\,\mathrm{Re}_{\kappa}^{0,6}(x) \quad (12b)$$

for a sphere, and

$$\mathrm{Nu} = \left(1 + \frac{2l}{d}\right)^{-1}\mathrm{Re}_{\kappa}^{0,46}(x) \quad (12c)$$

for a disk. The spread of the points around the functions (12) is 13% for a cylinder, 19% for a sphere, and 20% for a disk. The function (12a) yields values of Nu that are 13% higher than those form (11a). For a sphere, on the contrary, the function (12b) yields 20% lower Nu than does (11b). The experimental data obtained for a sphere and a cylinder can be considered to be in good agreement with the predictions from (11).

NOTATION

u, velocity; $|u_1|$, $|u_2|$, amplitudes of velocity components; ρ_0, gas density, r, x, radial and axial coordinates of the cylindrical coordinate system; t, time; ω, cyclic oscillation frequency; $Sh = \omega d/|u|$; $Re_{osc} = Ud/\nu$; d, characteristic size of the body (diameter of the sphere, cylinder, and disk); Pr, Prandtl number; $f = \omega/2\pi$; R, tube radius; R_0, half-width of the jet; $\bar{x} = x/R$; u_{00}, $|u_{10}|$, $|u_{20}|$, amplitudes at $x = x_0$; ℓ, cylinder length (disk thickness); c_0, speed of sound; $\bar{a}$, constant component; $\bar{b}$, $\bar{c}$, first and second harmonics; u_{0m}, $|u_{1m}|$, $|u_{2m}|$, amplitudes of velocity components at the jet axis; $\bar{u}_1 = u_1/u_{1m}$, $\bar{u}_1 = |u_1|/|u_{1m}|$, $u_2 = |u_2|/|u_{2m}|$; U(x), range of velocity oscillations at the jet axis.

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HYDRODYNAMIC CHARACTERISTICS OF ASCENDING GAS—LIQUID FLOW

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A new equation for a cell model of bubble flow is obtained on the basis of a previously unused boundary condition. A comprehensive investigation of the hydrodynamic characteristics of this flow has been carried out on a specially built apparatus, enabling us to test experimentally both the newly derived equation and that published earlier by Marrucci. A comparison of the calculated and experimental data showed that the Marrucci equation describes the bubble flow more accurately.

The extensive use of gas—liquid flows in power, chemical, and biological engineering supports the constant interest in research into the various characteristics of the individual phases and of the flow as a whole. The monographs [1-3] have generalized the results of such research. Nevertheless, the accuracy in calculating (in a theoretical approach) such characteristics as the gas content and the absolute and relative velocities of bubble ascent in mass bubbling still remains inadequate.

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